Victor Vinnikov
Ben Gurion University of the Negev

Linear matrix inequality representations of a convex set

Semidefinite programming is a powerful new technique that has revolutionized during the last decade optimization in general and the field of systems and control in particular. Semidefinite programming minimizes (or maximizes) a linear functional on a set descibed by a linear matrix inequality. This motivates an interesting mathematical problem: which sets can be described by a linear matrix inequality?

This problem leads to several different questions, depending on whether or not extra variables are admitted, and on whether the variables are scalars or (as in most applications in systems and control theory) matrices of a varying size. All of the questions are quite difficult, and the partial results available to date involve many different areas of mathematics: complex algebraic geometry (linear systems and line bundles on algebraic curves), real algebraic geometry (Positivstellensätze), and the newly emergent areas of free noncommutative algebraic geometry and function theory.

I will give an overview of some of these questions and results. The lecture will be accessible to graduate students.